

AD-A139 533

SOME THOUGHTS ON 3-D ISOPARAMETRIC ELEMENTS(U)
AERONAUTICAL RESEARCH LABS MELBOURNE (AUSTRALIA)
R JONES ET AL. JUN 83 ARL-STRUC-NOTE-489

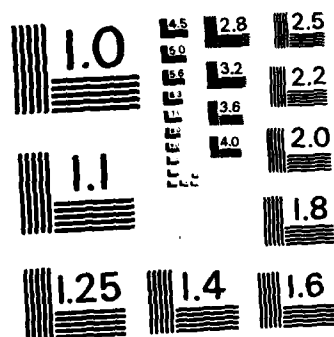
1/1

UNCLASSIFIED

F/G 20/11

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A



12

AD A139533

**DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORIES**

MELBOURNE, VICTORIA

STRUCTURES NOTE 489

SOME THOUGHTS ON 3-D ISOPARAMETRIC ELEMENTS

by

R. JONES and R. J. CALLINAN

Approved for Public Release

DTIC
ELECTE
MAR 30 1984
S A D

DTIC FILE COPY

AMERICAN STATES NATIONAL
TECHNICAL INFORMATION SERVICE
IS AUTHORIZED TO
REFERENCE AND SELL THIS REPORT

© COMMONWEALTH OF AUSTRALIA 1983

COPY No

JUNE 1983

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORIES

STRUCTURES NOTE 489

SOME THOUGHTS ON 3-D ISOPARAMETRIC ELEMENTS

by

R. JONES. and R. J. CALLIHAN

SUMMARY

With the increasing use of advanced fibre reinforced materials and the growing acceptance of damage tolerant design principles, three-dimensional finite element analysis is being used much more frequently. This paper presents some thoughts on the optimum use of three-dimensional isoparametric elements. Most commonly used element types are evaluated.



CONTENTS

	Page No.
1. INTRODUCTION	1
2. 3D ISOPARAMETRIC ELEMENTS	1
3. VIBRATION ANALYSIS	4
4. CRACK PATCHING	6
REFERENCES	
DISTRIBUTION	
DOCUMENT CONTROL DATA	



Accession	
NTIS ORIGIN	
NTIS TAB	
Document Number	
Project Number	
Page	
Page Count	
Author	
Title	
Subject	
Keywords	
Abstract	
Notes	
Comments	
Handwritten: A-1	

1. INTRODUCTION

In recent years, the damage tolerant design philosophy has been adopted by the aerospace industry. One requirement of this design philosophy is a detailed knowledge of the stress intensity factors along a surface flow. This knowledge can, in general, only be obtained by a detailed three-dimensional finite element analysis.

At the same time as damage tolerant design has become important, bonded repair technology (crack patching and strategic reinforcement) has also arisen as a highly cost effective method for extending the life of damaged components^[1,2] This procedure requires a detailed knowledge of the peel and shear stresses in the adhesive bond and, hence, often requires a full three-dimensional analysis. Since the adhesive thickness is usually very small, typically 0.1 mm, the elements used to model the adhesive often have very large aspect ratios with values of the order of 200 : 1 being common.

A similar problem arises in the study of impact damage in fibre composite laminates^[3]. In this case each ply, which is approximately 0.127 mm thick, needs to be modelled individually and the resulting elements have very high aspect ratios. As a result, the numerical solution to this problem is often highly ill-conditioned.

The present paper examines how each of these problems may best be tackled. Unfortunately, there is no single procedure which can be recommended generally.

2. 3D ISOPARAMETRIC ELEMENTS

For isoparametric elements, the shape function $N_i(\xi, \eta, \zeta)$ used to express the x, y, z coordinate system in terms of the curvilinear coordinate system, ξ, η, ζ are also used to define the displacement functions u, v and w within the element. Hence,

$$\mathbf{j} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = [IN_1, IN_2 \dots IN_p] \delta^e \quad (2.1)$$

$$= N \delta$$

and

$$\boldsymbol{\varphi} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = [IN_1, IN_2 \dots IN_p] \boldsymbol{\varphi}^e \quad (2.2)$$

$$= N \boldsymbol{\varphi}^e$$

Here

$$\delta^{eT} = (u_1, v_1, w_1, \dots, u_p, v_p, w_p) \quad (2.3)$$

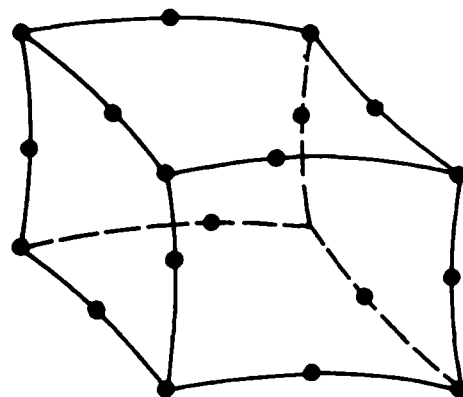
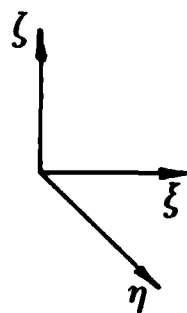
and

$$\boldsymbol{\varphi}^{eT} = (x_1, y_1, z_1, \dots, x_p, y_p, z_p) \quad (2.4)$$

where u_i, v_i and w_i are the displacements at the i th node which has coordinates x_i, y_i and z_i . The element is considered to have a total of p nodes and I is a 3×3 identity matrix. With this notation, the strain vector $\boldsymbol{\epsilon} = (\epsilon_x, \epsilon_y, \epsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})^T$ can be expressed in the form

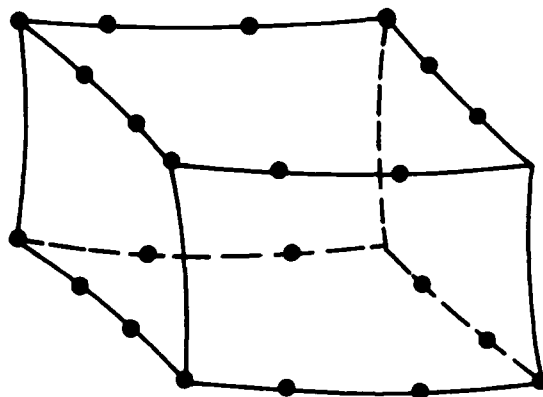
$$\boldsymbol{\epsilon} = B \delta^e \quad (2.5)$$

Interpolation functions



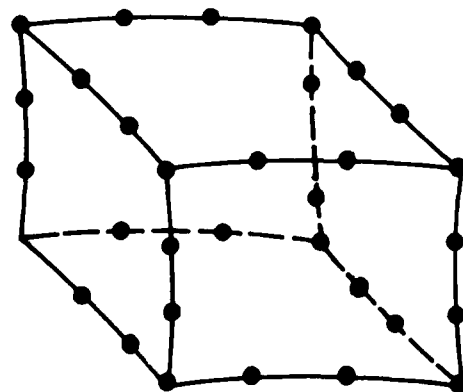
Quadratic in ξ, η, ζ

20 noded brick



Cubic in ξ, η
Linear in ζ

24 noded brick



Cubic in ξ, η, ζ

32 noded brick

FIG. 1 THREE DIMENSIONAL ISOPARAMETRIC BRICK ELEMENTS

so that the stiffness matrix k for the element can be written in the form

$$k = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 BDB^T |J| d\zeta d\eta d\xi \quad (2.6)$$

In equation (2.6), J is the Jacobian of the transformation and D is the elasticity matrix, i.e.

$$\begin{aligned} \sigma &= (\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx})^T \\ &= D\epsilon \end{aligned} \quad (2.7)$$

The integrals on the right hand side of equation (2.6) are evaluated using Gaussian quadrature. If the element has straight sides which intersect at right angles, then J is a constant. As a result, the order of the Gaussian quadrature which is required to integrate equation (2.6) exactly depends on the number of nodes and the curvature of the sides of the element; see Table 1 for the case when $J = \text{const.}$

TABLE 1
Integration Order Required

	Number of nodes			
	20	24	28	32
	3 × 3 × 3	4 × 4 × 2	4 × 4 × 3	4 × 4 × 4
Full integration				
Reduced integration	2 × 2 × 2	3 × 3 × 2	3 × 3 × 2	3 × 3 × 3

A number of papers have suggested that the optimum integration order is one order less than that required to integrate equation (2.6) exactly. This integration scheme is called reduced integration and provides an underestimate for k . It has also been suggested that reduced integration be used to evaluate the mass matrix

$$M = \int \int \int \rho N N^T |J| d\xi d\eta d\zeta \quad (2.8)$$

When using the twenty-noded brick elements to model plate bending problems, [4, 5] have shown that reduced integration gives very accurate results for aspect ratios a/t up to 1500/1, where t is the thickness of the plate and a is the length of the longest side of an element of the plate. Indeed [6] has shown that for two dimensional problems, reduced integration allows the accurate solution of problems involving the load transfer across a narrow interface.

In a recent paper,^[2] it was clearly shown that for vibration problems, the best results are obtained using reduced integration for the stiffness matrix and full integration for the mass matrix. This work was particularly interesting since both finite strip and finite element methods were considered.

The purpose of this paper is to investigate the accuracy of elements which have large aspect ratios. It should be mentioned that in this case, the accuracy of the results depends on the integration method adopted and on the word length of the computer used. The present investigation used double precision on a VAX 11/780.

3. VIBRATION ANALYSIS

We will begin our study by considering a thin, square plate of thickness t with sides of length b ($= 152.4$ mm). The edges of the plate are assumed to be simply supported and a total of thirty two elements of equal size is used to model one half of the plate. For this problem, the frequencies ω_{mn} are known to satisfy the following equation

$$\omega_{mn} b^2 \sqrt{\frac{12\rho(1-\nu^2)}{Et^3}} = (m^2 + n^2)\pi^2 / m^2 n^2 \quad (2.9)$$

where ρ is the density and E and ν are the Young's modulus and Poisson's ratio of the plate respectively. The errors in the numerically predicted frequencies are given in Table 2 for decreasing plate thickness, i.e. for increasing aspect ratio.

TABLE 2
Percentage error in frequencies

Mode	No. of nodes per element	Element aspect ratio											
		15			30			100			200		
		20	24	32	20	24	32	20	24	32	20	24	32
1, 1		0.4	10.5	0.1	0.1	11.1	0.6	7.2	4.8	4.8	0.3	33.1	10.7
1, 2		0.6	10.9	0.3	1.1	12.2	1.6	4.2	18.9	7.5	8.4	43.0	20.4
1, 3		5.7	7.2	1.8	2.7	17.8	9.2	30.1	40.4	28.9	57.9	84.9	58.8
2, 3		4.0	11.4	1.2	4.9	12.6	2.4	9.7	18.9	12.2	37.0	46.9	35.4
3, 3		10.4	19.6	9.9	26.7	26.2	16.7	4.9	11.3	18.2	80.9	9.2	63.1
1, 5		10.7	6.9	2.9	11.3	3.2	9.8	10.7	7.2	2.9	26.9	23.1	23.6

In this work, the stiffness matrix is evaluated using reduced integration whilst the mass matrix is evaluated using full integration. The matrices and the solution are done in double precision.

One reason for conducting this research was to evaluate the applicability of these elements in modelling delamination damage to fibre-composite laminates. Typically such damage has an inplane dimension between 25 and 50 mm, whilst the ply thicknesses are 0.127 mm. This results in elements with aspect ratios in the range of 50 : 1 to 200 : 1. Consequently the upper limit on the aspect ratios considered here is 200 : 1.

From these results we see that the twenty four noded element gives consistently bad results and, as a result, should not be used. For low values of aspect ratio, i.e. ≤ 15 , the thirty two noded element performs better than the twenty noded brick. However, this does not seem to be the case at higher aspect ratios.

This is in contrast to the results given in [5] where bending problems were considered and where the thirty two noded brick was accurate up to aspect ratios of 1500. This may be due to the computer on which the calculation was performed. Whilst the type of computer used is not specifically mentioned in [5], it is claimed that the results are accurate to twelve decimal places. This claim cannot be made in the present paper.

Although the results presented in Table 2 were calculated using full integration for the mass matrix, reduced integration was also investigated. When reduced integration was used to calculate the mass matrix, the errors were, in general, far greater and for the thirty two noded element, the assembled mass matrix was singular at aspect ratios of 100 : 1 and 200 : 1. Similarly the use of full integration to calculate the mass and the stiffness matrix resulted in frequencies which were far too high.



(a) Patched side of panel.
: patch radius 8 mm



(b) Unpatched side of panel.

FIG. 2

4. CRACK PATCHING

In recent years, the Aeronautical Research Laboratories, Australia, have pioneered the use of bonded fibre composite repairs to cracked metallic structures.^[1,2] In the analysis of this problem, the adhesive layer, typically 0.127 mm thick, the cracked sheet and the repair need to be modelled separately resulting in elements with aspect ratios in the range of 50 : 1 to 200 : 1.

In this section we will consider the repair of an edge crack in an aluminium sheet of dimensions 150 mm × 320 mm × 3.15 mm. The crack is 25.4 mm long and is repaired by a semi circular boron epoxy laminate with a radius of 80 mm and 0.889 mm thick and which has the fibres running perpendicular to the crack. The moduli of the laminate are

$$E_{11} = 208.3 \text{ GPa},$$

$$E_{22} = E_{33} = 25.4 \text{ GPa}$$

$$G_{13} = G_{23} = G_{12} = 7.24 \text{ GPa},$$

$$\nu_{13} = \nu_{12} = 0.183$$

$$\nu_{23} = 0.1667$$

The boron is bonded to the sheet using a low temperature cure paste adhesive with a Young's modulus $E_a = 1.89 \text{ GPa}$ and a shear modulus $G_a = 0.76 \text{ Pa}$. The adhesive thickness is nominally 0.165 mm thick, however since the adhesive used is a paste adhesive, its thickness is difficult to control and may vary up to 0.20 mm. The repaired panel was prevented from moving out of plane.

The model for this repair consisted of forty one of the twenty noded isoparametric bricks and fourteen of the fifteen noded isoparametric elements representing the sheet, whilst the adhesive and the repair each had twenty nine of the twenty noded bricks and thirteen of the fifteen noded elements. Reduced integration was used for each element type.

A clip gauge was used to measure the opening of the mouth of the crack and the strain in the patch at a point *B* near the crack tip was also measured (see figure 2). These measured quantities are compared in Table 3 with those predicted numerically.

TABLE 3

	Experiment	Analysis
Clip gauge opening	0.041 mm	0.032 mm
Strain in patch at <i>A</i>	1.30×10^{-3}	1.16×10^{-3}

Given the uncertainty in the assumed values for the adhesive thickness and moduli, the numerical results compare favourably with the experimental values. As a result, it appears that the use of reduced integration for elements with large aspect ratio is acceptable for this problem.

At this stage it must be mentioned that reduced integration does not always work. Indeed the authors have often encountered problems for which it gave erroneous results. In most of these cases, accurate results could be obtained by using directionally reduced integration, i.e. a $2 \times 2 \times 3$ Gauss quadrature for the twenty noded elements.

Unfortunately there appears to be no way of forecasting when reduced integration will fail. This situation is most unsatisfactory and when using reduced integration, the results should be carefully checked to ensure that spurious mechanisms are not present.

5. CONCLUSION

In this work we have seen that the twenty noded isoparametric element is unsuitable for use when the aspect ratios are large. We have also seen that, when using double precision and reduced integration, fairly accurate results can be obtained. However, it is clear that when modelling delamination damage in graphite epoxy laminates, the conditioning of the problem will improve if elements with large aspect ratios are avoided. To do this, a new super element is required which will allow an arbitrary grouping of plies within the element.

Following this work one such super element was developed at A.R.L. The next stage in this project is to use this new superelement in conjunction with the twenty noded brick to model simulated delamination damage in a graphite epoxy laminate and to compare the predicted surface strains with those measured experimentally.

REFERENCES

1. A. A. Baker. Australian Overview, repair of cracked or defective metallic aircraft components with advanced fibre composites, Proc. Workshop on Defence Applications of Advanced Repair Technology for Metal and Composite Structures, Naval Research Labs., Washington DC, July 1981, pp 11-35.
2. A. A. Baker, R. J. Callinan, M. J. Davis, R. Jones and J. G. Williams. Application of BFRP crack patching to Mirage III aircraft. Proc. ICCM3, Volume II, Paris, August 1980, pp 1424-1438 (1982).
3. R. Jones. Analysis of compression failures in fibre composite laminates, Proc. ICCM4, Volume 1, Tokyo, October 1982, pp 447-454 (1982).
4. O. C. Zienkiewicz. The Finite Element Method in Engng Science, McGraw Hill, London 1971.
5. G. C. Nayak and S. K. Agarwala. Simple modifications for mesh grading, unusual boundary conditions and slender element representations, Finite Element Methods in Engineering, Editors Y. K. Cheung and S. G. Hutton. Proc. 1976 International Conference on Finite Element Methods in Engineering, Adelaide, Australia, Dec. 1976, paper number 35, p. 35.1-35.18.
6. G. N. Pande and K. G. Sharma. On joint interface elements and associated problems of numerical ill conditioning, Int. J. Num. Methods in Engng, Volume 1, pp 293-300 (1979).
7. K. K. Teh, R. Jones and K. C. Brown. On the three-dimensional analysis of thick laminated plates. J. Sound and Vibration (in press).

DISTRIBUTION

AUSTRALIA

DEPARTMENT OF DEFENCE

Central Office

Chief Defence Scientist
Deputy Chief Defence Scientist
Superintendent, Science and Technology Programmes
Controller, Projects and Analytical Studies
Defence Science Representative (U.K.) (Doc. Data sheet only)
Counsellor, Defence Science (U.S.A.) (Doc. Data sheet only)
Defence Central Library
Document Exchange Centre, D.I.S.B. (17 copies)
Joint Intelligence Organisation
Librarian H Block, Victoria Barracks, Melbourne
Director General—Army Development (NSO) (4 copies)

(1 copy)

Aeronautical Research Laboratories

Director
Library
Superintendent—Structures

Divisional File—Structures
Authors: R. Jones
R. J. Callinan

Materials Research Laboratories

Director/Library

Defence Research Centre

Library

RAN Research Laboratory

Library

Navy Office

Navy Scientific Adviser

Army Office

Army Scientific Adviser

Air Force Office

Air Force Scientific Adviser
Aircraft Research and Development Unit
Scientific Flight Group
Library

Central Studies Establishment

Information Centre

DEPARTMENT OF DEFENCE SUPPORT

Government Aircraft Factories

Library

DEPARTMENT OF AVIATION

Library

STATUTORY AND STATE AUTHORITIES AND INDUSTRY

SEC of Victoria, Herman Research Laboratory, Library

B.H.P., Melbourne Research Laboratories

UNIVERSITIES AND COLLEGES

Adelaide	Barr Smith Library
Flinders	Library
Latrobe	Library
Melbourne	Engineering Library
Monash	Hargrave Library
Newcastle	Library
Sydney	Engineering Library
N.S.W.	Physical Sciences Library
Queensland	Library
Tasmania	Engineering Library
Western Australia	Library
R.M.I.T.	Library

CANADA

CAARC Coordinator Structures

NRC

Aeronautical and Mechanical Engineering Library

Universities and Colleges

Toronto Institute for Aerospace Studies

FRANCE

ONERA, Library

INDIA

CAARC Coordinator Structures

Defence Ministry, Aero Development Establishment, Library

Hindustan Aeronautics Ltd., Library

National Aeronautical Laboratory, Information Centre

ISRAEL

Technion-Israel Institute of Technology, Professor J. Singer

NETHERLANDS

National Aerospace Laboratory (NLR), Library

UNITED KINGDOM

CAARC, Secretary (NPL)

Royal Aircraft Establishment

Bedford, Library

Admiralty Marine Technology Establishment

St Leonard's Hill, Superintendent

National Physical Laboratory, Library

National Engineering Laboratory, Library

British Library, Lending Division

CAARC Co-ordinator, Structures

Universities and Colleges

Bristol	Engineering Library
Manchester	Professor, Applied Mathematics
Nottingham	Science Library
Southampton	Library
Strathclyde	Library
Cranfield Institute of Technology	Library

UNITED STATES OF AMERICA

NASA Scientific and Technical Information Facility
The John Crerar Library
Allis Chalmers Corporation, Library
Boeing Co.
Mr R. Watson

Lockheed-California Company
Lockheed Georgia

Universities and Colleges

John Hopkins	Professor S. Corrsin, Engineering
Massachusetts Inst. of Technology	M.I.T. Libraries

SPARES (10 copies)

TOTAL (97 copies)

Department of Defence Support
DOCUMENT CONTROL DATA

1. a. AR No. AR-002-962	1. b. Establishment No. ARL-STRUC-NOTE-489	2. Document Date June, 1983	3. Task No. DST 82/009
4. Title SOME THOUGHTS ON 3D ISOPARAMETRIC ELEMENTS		5. Security a. document Unclassified	6. No. Pages 13
		b. title c. abstract U U	7. No. Refs 7
8. Author(s) R. Jones and R. J. Callinan		9. Downgrading Instructions —	
10. Corporate Author and Address Aeronautical Research Laboratories P.O. Box 4331, MELBOURNE, VIC., 3001		11. Authority (as appropriate) a. Sponsor c. Downgrading b. Security d. Approval	
12. Secondary Distribution (of this document) Approved for public release Overseas enquirers outside stated limitations should be referred through ASDIS, Defence Information Services Branch, Department of Defence, Campbell Park, CANBERRA, ACT, 2601.			
13. a. This document may be ANNOUNCED in catalogues and awareness services available to . . . No limitations			
13. b. Citation for other purposes (i.e. casual announcement) may be (select) unrestricted (or) as for 13 a.			
14. Descriptors Finite element analysis Stress analysis Isoparametric elements Three-dimensional calculations			15. COSATI Group 1201
16. Abstract <i>With the increasing use of advanced fibre reinforced materials and the growing acceptance of damage tolerant design principles, three-dimensional finite element analysis is being used much more frequently. This paper presents some thoughts on the optimum use of three-dimensional isoparametric elements. Most commonly used element types are evaluated.</i>			

This page is to be used to record information which is required by the Establishment for its own use but which will not be added to the DISTIS data base unless specifically requested.

16. Abstract (Contd)		
17. Imprint Aeronautical Research Laboratories, Melbourne		
18. Document Series and Number Structures Note 489	19. Cost Code 216901	20. Type of Report and Period Covered —
21. Computer Programs Used —		
22. Establishment File Ref(s) —		

END

FILMED

4-84

DTIC